Dynamical Degeneracy of Relativistic Orbital Observables

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Abstract

Relativistic orbital observables are frequently interpreted as evidence for a unique underlying dynamical description. In particular, Mercury perihelion precession is commonly associated with relativistic geodesic motion and is often treated as a characteristic signature of spacetime geometry.

In this work we investigate the uniqueness of this inference through direct numerical construction. Several dynamically non-equivalent orbital models are calibrated to reproduce nearly identical relativistic-scale perihelion precession while exhibiting measurable differences in local temporal structure, orbital timing, phase-space evolution, Fourier composition, and sector traversal dynamics.

The numerical results demonstrate that agreement with a single integrated observable does not uniquely determine the underlying dynamics. Distinct orbital evolutions may generate observationally equivalent secular effects while remaining distinguishable through local diagnostics.

This establishes a form of dynamical degeneracy for relativistic orbital observables:

$$\text{same observable} \not\Rightarrow \text{same dynamics.}$$

The results do not challenge the internal consistency of General Relativity and do not propose an alternative theory of gravity. Instead, they highlight a broader methodological issue concerning the non-uniqueness of inverse dynamical reconstruction from integrated observables.

The work further motivates a distinction between geometric uniqueness and reconstruction uniqueness, and raises the question of whether successful reproduction of relativistic observables is sufficient to uniquely identify the underlying dynamical mechanism.

Keywords: orbital dynamics; perihelion precession; inverse problems; reconstruction degeneracy; dynamical non-equivalence; celestial mechanics; General Relativity; geodesic reconstruction; orbital observables; epistemology of physical models

Contents

1 Introduction

Relativistic orbital observables are often interpreted as evidence for a unique underlying dynamical description.

In particular, the anomalous perihelion precession of Mercury is conventionally associated with relativistic geodesic motion and is frequently presented as a characteristic signature of spacetime geometry.

Such interpretations implicitly assume that agreement between observation and prediction provides strong evidence for the uniqueness of the underlying dynamical mechanism.

However, this assumption is not generally guaranteed.

Integrated observables represent compressed summaries of orbital evolution accumulated over long temporal intervals. As a consequence, distinct local dynamical processes may produce identical integrated effects while differing substantially in their instantaneous evolution, temporal structure, phase-space trajectories, and orbital timing diagnostics.

The distinction between observable agreement and dynamical uniqueness is a general problem of inverse reconstruction. A successful prediction of an observable does not automatically imply uniqueness of the physical mechanism responsible for that prediction.

The present work investigates this question numerically using Mercury perihelion precession as a representative case.

Several dynamically non-equivalent orbital models are constructed and calibrated to reproduce nearly identical relativistic-scale perihelion advance. Despite agreement at the level of the integrated observable, the resulting trajectories exhibit measurable differences in local timing structure, phase-space evolution, Fourier composition, and orbital sector dynamics.

The purpose of this work is not to search for internal inconsistencies within General Relativity and not to propose an alternative theory of gravity.

Instead, the objective is methodological.

We investigate whether agreement with relativistic perihelion precession is sufficient to uniquely determine the underlying orbital dynamics, and more generally whether integrated relativistic observables should be interpreted as unique dynamical identifiers or as potentially degenerate outcomes of multiple distinct dynamical constructions.

The central question considered throughout this work is therefore:

$$\text{same observable} \implies ? \implies \text{same dynamics}.$$

The results suggest that this implication is not generally valid.

Data and Code Availability The numerical experiments reported in this work are fully reproducible. Source code, simulation scripts, and supporting materials are publicly available at:

`https://github.com/nekludoff/Publications/tree/main/Zenodo/Dynamical%20Degeneracy%20of%20Relativistic%20Orbital%20Observables`

2 Standard Relativistic Interpretation

In GR, test particle motion is described by geodesics:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0.$$

For weak-field nearly Keplerian motion, the resulting relativistic perihelion advance is approximately [1]

$$\Delta\varpi = \frac{6\pi GM}{a(1-e^2)c^2}.$$

The observed perihelion advance of Mercury is approximately

$$43 \text{ arcsec/century.}$$

Conventionally, successful agreement with this observable is interpreted as strong evidence supporting relativistic geometric dynamics.

3 Alternative Channel Dynamics

We consider alternative effective orbital dynamics of the form

$$\mathbf{a} = -\frac{\mu}{r^3}\mathbf{r} \cdot \mathcal{C},$$

where the channel factor $\mathcal{C}$ modifies the local response dynamics.

Several classes of channel models were explored numerically.

3.1 Radial channel

$$\mathcal{C} = 1 + \eta_1 \frac{\mu}{rc_*^2}.$$

3.2 Velocity channel

$$\mathcal{C} = 1 + \eta_2 \frac{v^2}{c_*^2}.$$

3.3 Anisotropic channel

$$\mathbf{a} = -\frac{\mu}{r^3}\mathbf{r} + \eta_4 \frac{\mu}{r^2 c_*^2} v_r \mathbf{v}.$$

These models are dynamically non-equivalent, yet can be calibrated to reproduce nearly identical perihelion precession.

4 Definition of Dynamical Non-Equivalence

The central claim of this work is not merely the existence of alternative equations of motion, but the existence of dynamically non-equivalent orbital models producing the same integrated observable.

Let

$$D_i, \ D_j$$

denote two orbital dynamical systems.

We define them as dynamically equivalent if all local orbital diagnostics coincide:

$$Q_k[D_i] = Q_k[D_j]$$

for every diagnostic functional Q_k.

Examples include:

$$r(\theta), \qquad v_r(\theta), \qquad v_t(\theta), \qquad t(\theta), \qquad \mathcal{F}[r(\theta)].$$

Two models are called dynamically non-equivalent if there exists at least one local diagnostic such that

$$Q_k[D_i] \neq Q_k[D_j].$$

At the same time, the models may remain observationally equivalent with respect to a selected integrated observable:

$$\mathcal{O}[D_i] = \mathcal{O}[D_j].$$

In the present work

$$\mathcal{O} = \Delta\varpi.$$

The existence of such pairs

$$D_i \neq D_j, \qquad \mathcal{O}[D_i] = \mathcal{O}[D_j]$$

constitutes dynamical degeneracy.

$$\begin{tikzcd}
 & D \arrow[r] \arrow[dr] & \{Q_k\} \arrow[d] \\
\mathcal{O} & & \{r(\theta), v_r(\theta), v_t(\theta), t(\theta), \mathcal{F}[r]\}
\end{tikzcd}$$

Figure 1: Mapping from orbital dynamics to local diagnostics and integrated observables. Different dynamical systems may produce identical integrated observables while remaining distinguishable through local diagnostics.

5 Numerical Experiments

Numerical integration was performed using high-order adaptive integration (DOP853) over multiple Mercury orbital periods.

The calibrated models produced nearly identical perihelion advance:

$$\Delta\varpi_{\text{radial}} = 42.9808''/\text{century},$$

$$\Delta\varpi_{\text{velocity}} = 42.9807''/\text{century},$$

$$\Delta\varpi_{\text{anisotropic}} = 42.9808''/\text{century}.$$

Despite this agreement, substantial differences were observed in local orbital dynamics.

- orbital period,
- local velocity profiles,
- phase-space evolution,
- Fourier structure of $r(\theta)$,
- sector traversal times.

5.1 Sector traversal time experiment

The orbit was partitioned into equal angular sectors.

Traversal times for individual sectors were computed between consecutive perihelia.

Although the integrated perihelion precession remained nearly identical across models, local traversal times differed systematically.

For example:

$$\Delta t_{\max} \sim 0.3\text{–}0.7 \text{ seconds}$$

between dynamically distinct models calibrated to identical precession.

This demonstrates that identical integrated relativistic observables do not uniquely determine local orbital dynamics.

$$D_1 \qquad D_2 \qquad D_3$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\Delta\varpi = 43''/\text{century}$$

Figure 2: Multiple dynamically non-equivalent orbital models producing the same integrated observable.

Model	Precession	Max Sector Shift	Period Shift	Fourier Difference
radial_eta1	43”	0	0	0
speed_eta2	43”	0.32 s	1.61 s	non-zero
anisotropic4	43”	0.66 s	3.91 s	non-zero

Table 1: Examples of dynamically non-equivalent models reproducing the same perihelion precession.

5.2 Two-Body Channel Extension

As a preliminary extension, a simplified Sun–Mars–Mercury channel model was also tested.

The solar channel was parameterized by

$$\eta_1 = 6.0, \qquad \eta_2 = \eta_3 = \eta_4 = 0,$$

while the Mars channel used

$$\eta_1 = 1.0, \qquad \eta_2 = \eta_3 = \eta_4 = 0.$$

The resulting mean perihelion precession was

$$\Delta\varpi = 44.5759''/\text{century},$$

with a substantially larger orbital scatter than in the single-source calibrated models.

This preliminary result suggests that multi-body channel contributions can modify both the mean integrated observable and its orbit-to-orbit variability.

We therefore treat inter-body channel modulation as a separate problem rather than as part of the present single-observable degeneracy construction.

6 Observable Degeneracy

The numerical experiments demonstrate that the mapping

$$D \longrightarrow \Delta\varpi$$

is not injective.

Multiple dynamically distinct orbital models produce the same perihelion precession observable.

Formally,

$$D_i \neq D_j$$

does not imply

$$\Delta\varpi_i \neq \Delta\varpi_j.$$

Instead, there exist families of models satisfying

$$\Delta\varpi_i = \Delta\varpi_j$$

while differing in

$$t(\theta), \quad r(\theta), \quad v_r(\theta), \quad v_t(\theta), \quad \mathcal{F}[r(\theta)].$$

This establishes a non-uniqueness of dynamical reconstruction from integrated orbital observables.

7 Geometric Uniqueness versus Observational Non-Uniqueness

Within General Relativity, once the metric tensor

$$g_{\mu\nu}$$

and initial conditions are fixed, geodesic motion is uniquely determined.

Symbolically,

$$(g_{\mu\nu}, x_0, u_0) \Longrightarrow \gamma(\tau).$$

Thus GR provides uniqueness of trajectory inside the chosen geometric framework.

However, the inverse problem is fundamentally different.

An observed perihelion advance

$$\Delta\varpi$$

does not uniquely determine

$$g_{\mu\nu}$$

nor the underlying dynamical structure.

The numerical constructions presented in this work explicitly demonstrate that distinct local dynamics may reproduce the same integrated orbital observable.

Consequently,

$$\Delta\varpi \not\Rightarrow \gamma(\tau).$$

Uniqueness inside a geometric formalism should therefore not be confused with uniqueness of dynamical reconstruction from observations.

8 Discriminating Observables

Not all observables possess the same discriminating power.

Integrated observables such as perihelion precession may be dynamically degenerate.

Local observables are expected to provide stronger discrimination.

Examples include:

- orbital period,
- sector traversal times,
- Doppler observables,
- ranging residuals,
- Fourier spectrum of $r(\theta)$,
- phase-space diagnostics.

Future observational tests should therefore focus on quantities sensitive to local temporal structure rather than solely integrated geometric effects.

9 Historical Information and Geometric Reconstruction

The preceding discussion raises a more general question.

Many physical systems evolve through the cumulative action of numerous interactions over extended periods of time.

In orbital mechanics, the actual trajectory of a body may depend on:

- long-term perturbations,
- multi-body interactions,
- resonances,
- environmental effects,
- and accumulated historical evolution.

From this perspective, the observed state of a system may contain information resulting from a long dynamical history.

General Relativity approaches orbital motion differently.

Once a spacetime geometry

$$g_{\mu\nu}$$

and initial conditions are specified, the trajectory is obtained from local geodesic evolution.

Symbolically,

$$(g_{\mu\nu}, x_0, u_0) \Longrightarrow \gamma(\tau).$$

This raises an epistemological question.

If observable orbital effects partly reflect accumulated historical dynamics, by what mechanism is such information represented within a local geometric description?

More specifically, if long-term dynamical history contributes to present observables, then one may ask whether the geometric structure itself should be interpreted as:

1. a fundamental physical entity;
2. an effective representation of accumulated dynamics;
3. or a compressed description of historically evolved orbital behaviour.

The present work does not attempt to answer this question.

Rather, it highlights the existence of dynamically distinct models producing the same integrated observable and motivates further investigation of the relation between historical dynamics and geometric reconstruction.

9.1 Available History versus Physical History

The notion of historical information should be interpreted with care.

Classical history-dependent dynamical systems are often written in forms such as

$$\dot{x}(t) = F(x(t), x(t-\tau))$$

or

$$\dot{x}(t) = F\left(x(t), \int_0^t K(t-s)x(s)\,ds\right).$$

In such formulations, historical information enters the equations explicitly.

However, a deeper reconstruction problem remains.

To compute the future evolution, one must know the relevant history. Yet that history is itself the result of earlier dynamics, which in turn depends on even earlier states. Formally, one obtains a recursive dependence

$$\text{future} \leftarrow \text{history} \leftarrow \text{earlier history} \leftarrow \cdots$$

In realistic physical systems, the complete physical history is never available.

Consequently, practical reconstruction always replaces the unknown physical history by an available approximation

$$H \;\to\; \hat{H},$$

or equivalently

$$I_H \;\to\; \hat{I}_H.$$

The distinction is important.

The physical history

$$H$$

represents the complete set of interactions, perturbations, and influences that contributed to the present state of the system.

The reconstructed history

$$\hat{H}$$

represents only the finite subset of information available to the observer or incorporated into the model.

In general,

$$H \neq \hat{H}.$$

Thus, practical prediction is never performed using the full physical history of the system. Instead, prediction is performed using a reconstructed approximation of that history.

This observation motivates a broader methodological question.

If both Newtonian and geometric reconstructions ultimately operate on the same reconstructed information

$$\hat{H},$$

then the advantage of geometric reconstruction cannot be assumed a priori.

One may therefore ask whether geometric formulations provide additional physical information, increased predictive power, or primarily an alternative organization of information already present in the reconstructed dynamical description.

The present work does not attempt to answer this question. It merely identifies the distinction between physical history and reconstructed history as a potentially important aspect of orbital reconstruction.

10 Reconstruction Dependence of Physical Geodesics

The discussion above suggests a distinction between mathematical and physical geodesics.

Let

$$\gamma(g_{\mu\nu}, I)$$

denote a geodesic obtained from a spacetime metric $g_{\mu\nu}$ and a set of input data I, including masses, orbital parameters, boundary conditions, and other model assumptions.

Observation

Suppose that refinement of the physical model leads to an updated dataset

$$I \to I'.$$

If

$$\gamma(g_{\mu\nu}, I) \neq \gamma(g_{\mu\nu}, I'),$$

then the physically relevant geodesic depends not only on the geometry itself, but also on the reconstruction procedure used to obtain the physical model.

Consequently, the observed geodesic becomes a function of both

$$g_{\mu\nu}$$

and

$$I.$$

In this sense, physically relevant geodesics may be reconstruction-dependent.

Mathematical versus Physical Geodesics

For a fixed metric tensor, geodesic motion is uniquely determined:

$$g_{\mu\nu} \Longrightarrow \gamma.$$

This may be called a mathematical geodesic.

However, practical orbital reconstruction involves an additional layer:

$$\text{Universe} \Longrightarrow (g_{\mu\nu}, I) \Longrightarrow \gamma.$$

The first step is not given directly by observation and is subject to continual refinement as new measurements become available.

Therefore, while mathematical geodesics are unique within a fixed geometric framework, physically reconstructed geodesics may evolve together with the underlying model of the system.

This distinction raises a methodological question.

If physically relevant geodesics require repeated refinement of the global reconstruction of the system, in what sense should they be regarded as more fundamental than trajectories obtained through conventional dynamical integration?

The present work does not attempt to answer this question, but identifies it as a consequence of the distinction between geometric uniqueness and reconstruction dependence.

11 Information Content and Geometric Reconstruction

The preceding discussion motivates a related question concerning the informational role of geometric descriptions.

In practical orbital mechanics, increasing predictive accuracy requires progressively more detailed information about the physical system.

For conventional dynamical models, this information includes masses, orbital elements, perturbing bodies, resonances, multipole moments, tidal effects, and other contributions affecting long-term evolution.

Geometric reconstruction appears conceptually different. Orbital motion is represented through spacetime geometry and geodesic evolution rather than through explicit accumulation of pairwise interactions.

However, in practical applications, increasingly accurate geodesic predictions also require increasingly detailed information about the physical system. The metric reconstruction itself depends on masses, distributions of matter, perturbations, boundary conditions, and observational refinements.

This observation suggests an important distinction between representational compression and informational reduction.

A geometric formulation may reorganize information into a compact mathematical structure without necessarily reducing the amount of information required for accurate prediction.

Symbolically, one may ask whether

$$\text{Geometry} = \text{Additional Physical Information}$$

or merely

$$\text{Geometry} = \text{Alternative Organization of Information}.$$

The numerical constructions presented in this work reinforce this question.

A relativistic-scale perihelion precession

$$\Delta\varpi \approx 43''/\text{century}$$

was reproduced using several dynamically non-equivalent channel models containing only a highly simplified representation of the physical system.

The existence of such models suggests that a successful prediction of a particular integrated observable does not uniquely identify the informational structure responsible for that prediction.

This leads to a broader methodological question.

If both geometric and dynamical approaches require progressively more complete knowledge of the physical system as observational precision increases, then where exactly does the additional explanatory content of geometric reconstruction reside?

More generally, one may distinguish between

Predictive Superiority

and

Representational Superiority.

A model may provide a compact and elegant representation of observations while deriving its predictive power from information already contained in the underlying dynamical description.

The present work does not attempt to resolve this question. Rather, it identifies it as a natural consequence of observable degeneracy and reconstruction dependence.

12 Scope and Limitations

The dynamical models considered in this work represent physically well-defined orbital systems. They preserve dimensional consistency, admit stable numerical integration, and constitute valid modifications of central-force dynamics. No assumption of non-physical behavior is introduced.

The scope of the present study is limited in the following methodological sense.

Single Observable Focus

The analysis concerns a single integrated observable, namely perihelion precession.

The results establish non-uniqueness of dynamical reconstruction with respect to this observable alone. No claim is made regarding degeneracy of all relativistic observables or all possible gravitational measurements.

Constructive Demonstration Rather Than Exhaustive Classification

The channel models introduced in this work serve as explicit constructive examples of dynamically non-equivalent orbital systems producing observationally equivalent secular effects.

The objective is to demonstrate existence rather than to provide a complete classification of all possible degenerate dynamical constructions.

Single-Source Calibration

The primary numerical experiments focus on a dominant central gravitational source.

While preliminary investigations indicate that multi-body channel interactions may introduce additional phase-dependent modulation effects, such phenomena are beyond the scope of the present work and will be investigated separately.

These limitations do not restrict the physical validity of the constructed models. Rather, they define the domain of the specific claim established here:

$$\text{agreement with a single integrated observable;} \not\Rightarrow \text{; uniqueness of the underlying dynamics.}$$

Accordingly, the present work should be viewed as a study of reconstruction degeneracy in orbital dynamics rather than as a critique of any particular gravitational theory.

13 Conclusion

The present work introduced several dynamically non-equivalent orbital constructions that reproduce essentially identical relativistic-scale perihelion precession while exhibiting measurable differences in local temporal structure, phase-space evolution, Fourier composition, and orbital timing diagnostics.

The numerical experiments demonstrate that agreement with a single integrated observable,

$$\Delta\varpi,$$

does not uniquely determine the underlying dynamical mechanism.

More generally, the mapping

$$D \to O$$

from orbital dynamics (D) to integrated observables (O) need not be injective. Distinct dynamical systems may therefore generate observationally equivalent secular effects while remaining distinguishable through local diagnostics.

The results suggest that relativistic perihelion precession should be regarded as a low-dimensional integrated observable rather than as a unique identifier of orbital dynamics.

This observation motivates a broader methodological distinction between geometric uniqueness and reconstruction uniqueness. A uniquely defined geodesic within a given geometric framework does not necessarily imply uniqueness of the underlying physical reconstruction capable of producing the same observable outcome.

The work further raises an informational question. If increasingly accurate orbital predictions require progressively more detailed knowledge of the physical system, then it remains an open problem whether geometric reconstruction provides additional physical information or primarily reorganizes information already contained in the underlying dynamical description.

The present study is intentionally limited to perihelion precession and does not claim non-uniqueness of all relativistic observables. Rather, it establishes the existence of dynamically distinct constructions reproducing the same integrated orbital observable.

Preliminary multi-body channel experiments indicate that additional bodies may introduce phase-dependent modulation of orbital observables in addition to secular corrections. These effects suggest a broader class of inter-body reconstruction phenomena and will be investigated separately.

Future work may therefore proceed in two directions:

1. identification of observational discriminators capable of distinguishing dynamically degenerate orbital models, including ranging residuals, Doppler observables, timing observables, and local phase-space diagnostics;

2. investigation of multi-body channel interactions and phase-dependent modulation effects in systems containing multiple gravitational sources.

The central conclusion remains unchanged:

$$\text{same observable} \not\Rightarrow \text{same dynamics.}$$

Consequently, successful reproduction of perihelion precession alone cannot establish the uniqueness of the underlying dynamical description.

A Channel Models Used in Numerical Experiments

A.1 Model A

$$a = -\frac{\mu}{r^3}\mathbf{r}\left(1 + \eta_1 \frac{\mu}{rc^2}\right)$$

A.2 Model B

$$a = -\frac{\mu}{r^3}\mathbf{r}\left(1 + \eta_2 \frac{v^2}{c^2}\right)$$

A.3 Model C

$$\mathbf{a}_C = -\frac{\mu}{r^3}\mathbf{r} + \eta_4 \frac{\mu}{r^2 c^2} v_r \mathbf{v}.$$

A.4 Common Model

$$\mathbf{a} = -\frac{\mu}{r^3}\mathbf{r}\left(1 + \eta_1 \frac{\mu}{rc^2} + \eta_2 \frac{v^2}{c^2} + \eta_3 \frac{v_r^2}{c^2}\right) + \eta_4 \frac{\mu}{r^2 c^2} v_r \mathbf{v}.$$

The models were selected to represent radial, velocity-dependent, and anisotropic channel corrections. No claim is made that these forms are physically fundamental. They serve only as explicit examples of dynamically distinct constructions producing identical integrated observables.

The purpose of these constructions is existence proof rather than physical interpretation.

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